On the General Current Dependence of the Distributed Series Resistance of Solar Cells: The Influence of the Base Resistivity

Jan-Martin Wagner, Jürgen Carstensen, and Rainer Adelung

Abstract. The lumped series resistance $R_s$ of a silicon solar cell isn’t constant but depends on the operating point of the solar cell. For describing the relevant current dependence analytically, only few theories exist that can easily be applied to experiments. These are: (i) the ad-hoc modelling by Araújo et al., modified by Breitenstein et al., and (ii) the LR-$R_s$ approach by Wagner et al. Both have fundamental limitations: Whereas Araújo’s ad-hoc model is partly unphysical, the LR-$R_s$ approach misses the influence of the base resistivity. Here, we discuss these shortcomings, and we show where to include the base resistivity in the LR-$R_s$ approach. In addition, we discuss the current dependence of the lumped series resistance of a solar cell in general terms, thereby clarifying the underlying physical basics.

Keywords: Distributed Series Resistance, Lumped Series Resistance, Current-Dependent Series Resistance, Joule Losses, Linear Response, Equivalent Circuit

1. Introduction

In general, an ohmic resistor is a perfectly linear circuit element: It has a constant value, and it is fully symmetric, i.e. its value neither depends on the strength nor on the direction of the current that flows through it. Although this seems highly trivial, it is nevertheless highly relevant when it comes to solar cells, since it is long-since known that the lumped series resistance, $R_s$, of a p–n junction solar cell isn’t constant but depends on the operating point of the solar cell, i.e. it varies with the illumination intensity and the loading condition; for relevant measurements, see, e.g., [1–5]. Does that mean that the lumped series resistance of a solar cell is intrinsically non-ohmic? Although it seems that the answer to this question is clear – namely, that $R_s$ is indeed ohmic (i.e., linear), but due to the distributed nature of the series resistance, the lumped value depends on the current flow pattern throughout the solar cell –, there are nevertheless some works in the literature that explicitly declare $R_s$ to be nonlinear or, equivalently, to depend on the strength of the current flowing through it [6–12], or at least on the direction of that current [13]. Obviously, this calls for a closer look into the subject matter.

From the theoretical side, the effect that $R_s$ depends on the operating point has been analyzed both analytically (cf., e.g., [3, 14–16]) and numerically (cf., e.g., [3, 14, 17, 18]), with the basic finding that “the cause of the decrease in series resistance with increasing forward bias lies in shorter paths for current available at higher junction voltages.” [15] The full analytical solutions for this effect are rather demanding, since the relevant equations are quite intricate. (There is one analytical and numerical work [7] which, together with the related experimental and quasi-analytical work presented in [11], needs special attention since it is built on a non-linear $R_s$ definition; the relevant discussion will be the main part of the present work.) On the
other hand, both by detailed measurements and in the analytic linear-response series resistance (LR-\(R_s\)) concept [19–21] we have found that this effect (\(R_s\) depending on the operating point) could be described rather simply by including the effective forward-bias resistance of the p–n junction, i.e., the diode resistance \(R_D\), in the series resistance, \(R_s\), of the standard equivalent circuit. This is achieved in the following way: \(R_D\) is defined as the inverse of the slope of the diode’s \(I–V\) characteristic, i.e., \(R_D^{-1} = dI_D/dV_D\), with the forward-bias diode current \(I_D\) (also known as the dark current), and the internal diode voltage \(V_D\); in the single-diode case, \(R_D^{-1} \approx I_D / V_{th}\), with the thermal voltage \(V_{th} = kT / q\). In the LR-\(R_s\) concept, the total lumped and current-dependent series resistance can be written as \(R_s(I_D) = R_{s,\text{distr}}(I_D) + R_{s,\text{non-distr}}\). The distributed part, comprising the current dependence, consists of a parallel circuit of the effective diode resistance, weighted by a certain factor \(g\), and a fixed series resistance \(R_{s,\infty}\), giving the value of \(R_{s,\text{distr}}\) for infinite diode resistance:

\[
\frac{1}{R_{s,\text{distr}}(I_D)} = \frac{1}{R_{s,\infty}} + g \frac{1}{R_D} \approx \frac{1}{R_{s,\infty}} + g \frac{I_D}{V_{th}}
\]

(1)

This formula is special for two reasons: First, it is a mathematically rather simple expression compared to other published theoretical results (cf., e.g., [15]). This is due to Eq. (1) not being a solution to the full problem but only to a linearized version (for details see [20, 22]). Second, although the series resistance is known to vary with illumination intensity and the loading condition, in Eq. (1) only the forward-bias dark current \(I_D\) appears. Note, however, that for any solar cell this dark current is related to the photocurrent, \(I_{ph}\), and the external current, \(I_{ext}\): here, we take the dark current to have the positive sign, so the relevant relation reads

\[
I_{ext} = I_D - I_{ph}
\]

(2)

Here, \(I_{ph}\) is the absolute value of the photocurrent. For this sign convention we have that under forward bias in the dark, \(I_D = I_{ext} > 0\), at short circuit, \(I_{ext} = -I_{ph} < 0\) as well as \(I_D = 0\), and at open circuit, \(I_{ext} = 0 = I_D - I_{ph}\), so \(I_D = I_{ph} > 0\). Hence, although in the LR-\(R_s\) concept only \(I_D\) appears explicitly in the equations, nevertheless the dependencies of \(R_s\) on the other currents, \(I_{ext}\) and \(I_{ph}\), are included in this theory as well, namely by means of Eq. (2).

In Eq. (1), the constant \(R_{s,\infty}\) is identical to the standard text-book expression for the effective geometrical series resistance contribution of emitter sheet and grid, \(R_{s,\infty} = \frac{1}{2} (\rho_{sh} d/b + 2 r_F b) \times bd / A_{cell}\) (2d: busbar distance, 2d: grid finger distance, \(A_{cell}\): total cell area, \(r_F\): finger line resistivity in \(\Omega/mm\), \(\rho_{sh}\): emitter sheet resistivity in \(\Omega/cm\)) [20, 23]. From the LR-\(R_s\) theory, we derived the value of the weighting factor \(g\) in Eq. (1) to be \(\frac{3}{2}\) for a rectangular [20] and 1 for a circular geometry [21] of the grid. But determining it from our measurements reported in [19], where indeed a linear relation according to Eq. (1) has been found, we however find a slope giving \(g = 1.8\) for the H-type grid cell investigated there.

The effect of \(R_{s,\text{distr}}\) on the \(I–V\) characteristic was treated both analytically and numerically by Araújo et al. [7] for a simple 2D model geometry where the current flow in the base was taken as strictly vertical and that in the emitter as strictly horizontal. With an ad-hoc model for the lumped series resistance, the latter’s behavior under various loading conditions was obtained. Breitenstein et al. empirically modified the resulting relations and successfully applied them to measurements [11]. However, we have shown in [19] that the numerical results presented in [7] are inconsistent with a model-free treatment of the lumped series resistance (based on the actual Joule losses; see also below) – but only partially: The general way how \(R_s\) decreases in dependence on the dark diode current is nevertheless quite similar in all cases.

All these facts, both the (partial) successes and the (partial) failures, call for a deeper theoretical analysis of \(R_{s,\text{distr}}(I_D)\). This is done here by means of numerical simulations of various loading conditions for the same 2D model geometry as used in [7].
2. Theoretical and numerical approach

In general, a lumped series resistance $R_s$ of a solar cell can be defined in two fundamentally different ways, either from voltage losses or from ohmic heating (integrated Joule losses). Yet only the latter way is of general validity, because it is model-free [21]. Hence, to serve as a reference, we determine $R_s$ from the resistive Joule losses by setting $P_{res} = \int \rho J \, d\tau = R_s I_{ext}^2$. These results are compared with the analytical results obtained from the LR-$R_s$ concept [20, 21] and with those stemming from the ad-hoc model and its empirical modification [7, 11].

We use the same geometry and model parameters as Araújo et al. [7]. Accordingly, all material parameters — the emitter and base bulk resistivities, $\rho_e$ and $\rho_b$, the saturation current density, $J_0$, and the photocurrent, $J_{ph}$ — are taken to be homogeneous, only a single diode is used in the modeling, and shunts are neglected. The base thickness $w_b$ is combined with the base resistivity in the parameter $r_b = \rho_b w_b$, and for the emitter the parameter is $r_e = \rho_e d^2 / w_e$, with the emitter thickness $w_e$, as above, $d$ is half the grid finger distance. These parameters are related to the ones used by Breitenstein et al. [11] as follows (for consistency w.r.t. the units of measurement, they are written here also with small letters): $r_{dis} = r_d / 3$, $r_{nom} = r_b$, and the total lumped series resistance at zero dark current is given by $r_s,0 = r_{dis} + r_{nom}$. To relate them to the LR-$R_s$ parameters, the total area $A_{cell}$ of the solar cell is relevant: From Eq. (15) of [11] for $J_{ext} \to \infty$ one has that $R_{non-distr} = (3 r_{dis} r_{nom})^{1/6} / A_{cell}$, so that $r_{s,\infty} = r_{s,0} / A_{cell} - R_{non-distr}$.

For our calculations (done with MATLAB), we numerically solve the differential equation for the lateral voltage and current distribution, with the vertical current being given analytically (single-diode model plus photocurrent). For the dark case, a set of prescribed voltages at the external contact was used. Their values are irrelevant, because only the resulting total dark current is needed; the latter is obtained from the numerical integration. Finally, we use Eq. (18) of [7] to also determine the corresponding Araújo model’s $r_{s,\infty}$ value. For the open-circuit case, the Joule losses cannot be determined directly since there is no external current flowing. So, a similar approach was chosen as in [7] where the $J$–$V$ behavior around $V_{oc}$ was considered: For a prescribed photocurrent, we first determine the relevant open-circuit voltage analytically, then we choose a few sampling points below and above $V_{oc}$ for which we numerically solve the differential equation as before. The thereby-obtained Joule-$r_s$ data are interpolated by a spline, and the final Joule-$r_s$ result is obtained from evaluating this spline at $V_{oc}$. The value of $r_{s,\infty}$ in Araújo’s model, as given by Eq. (20) of [7], is determined completely analytically.

3. Results and discussion

For our simulations, we use the same parameter values as in [7], i.e., $r_e = 45 \text{ m\Omega cm}^2$ and $r_b = 7.5 \text{ m\Omega cm}^2$, as well as $V_{th} = 25 \text{ mV}$ and $J_0 = 1.25 \times 10^{-12} \text{ A/cm}^2$ [24]. In Fig. 1 we show the open-circuit and the dark case, analogous to Fig. 4 of [7]. Since in the open-circuit case the full photocurrent flows as forward-bias diode current, $J_0$, the latter can serve as abscissa for all cases treated in Fig. 1. As already noted in [19], the dark and the open-circuit Joule-$R_s$ results agree nearly completely. However, none of the model curves, neither those of Araújo et al. [7] nor those from the LR-$R_s$ concept [20, 21] (with LR-$R_{s,1}$ having $g = 1$ and LR-$R_{s,2}$ having $g = \frac{1}{2}$), agrees with the Joule-$R_s$ results. Nevertheless, the limiting values of the Joule-$r_s$ for very low and very high currents are met by the model of Araújo et al. – which is important as it justifies the above relations for $R_{non-distr}$ and $R_{s,\infty}$; only by this it is possible to plot the analytically given LR-$R_s$ curves in a meaningful way.

The Joule results for the dark and the open-circuit cases being equal is in contrast not only to Araújo’s model curves: That the dark resistance is always the lowest one has previously been claimed also explicitly by other authors (cf., e.g., [8, 9, 13]). Yet by luminescence-based series resistance measurements we have found that this doesn’t hold true: When illumination conditions are chosen with a dark diode current identical to that of the dark case, the series resistance is the same for both cases; this holds for all illumination conditions [19, 25].
Figure 1. Lumped series resistance in dependence on the forward-bias diode current: Open-circuit and dark case as in [7], determined from the Joule losses, the Araújo ad-hoc model, and according to the LR-\( R_s\) concept (LR-\( R_{s,1} \): \( g = 1 \); LR-\( R_{s,2} \): \( g = \frac{1}{2} \)).

So, where do the discrepancies come from? Remembering the deviation of the theoretical \( g \) values from the experimentally determined one (as mentioned in the introduction), it is not surprising that both LR-\( R_s\) curves do not agree with the Joule behavior; a further discussion will be given below. The deviating behavior of Araújo’s model curves was attributed in [19] to the ad-hoc modelling of \( R_s \) being dependent on \( J_{ext} \) and on \( J_{ext} \) only [7], making it a nonlinear element. This nonlinearity means that such an \( R_s \) is intrinsically non-ohmic, which is unphysical. This conclusion is supported by the fact that even the dark case, where the external current equals the diode current, is not correctly represented by Araújo’s ad-hoc model.

Figure 2. Lumped series resistance in dependence on the forward-bias diode current. (a) Dark cases as in Fig. 1; in addition, the “empirical dark” curve of Breitenstein et al. [11] and LR-\( R_{s,3} \) with \( g = 6 \) are shown. (b) Dark cases as in (a), but for vanishing base resistivity.

In Fig. 2(a) we compare the dark cases of Fig. 1 with the “empirical dark” curve of Breitenstein et al. [11] and the LR-\( R_s \) result with \( g = 6 \). In their work, Breitenstein et al. had noted that the analytically-given \( R_s^{\text{oc}} \) result of Araújo et al. looks similar to the non-analytical \( r_s^{\text{dark}} \) one, just slightly shifted; and they found that empirically rescaling the current in the analytical expression of the former, they obtained a rather good representation of the latter [11]. Similarly, since increasing \( g \) from \( \frac{1}{2} \) to 1 leads to a shift of the LR-\( R_s \) model curve to the left, we try to use \( g \) as fitting parameter, and as shown in Fig. 2(a), for \( g = 6 \) we find a very good agreement with the onset of the decrease of the Joule-\( r_s^{\text{dark}} \) curve.

From Fig. 2(a), two aspects arise: On the one hand, since also in the experiment a \( g \) value larger than 1 was obtained, a deeper understanding is needed for such values of \( g \) (see below);
on the other hand, the deviation between the modeling of [7, 11] and the Joule-$r_s$\text{dark} curve, starting just a little after the onset of the decrease of the latter, leads us to questioning the success reported in [11]. However, Fig. 2(a) also shows that the deviation is small and that the general shape of the decrease of the series resistance is the same for all four relevant curves, which might explain why not only passed the error unnoticed, but also why it was ignored that the underlying model in [7] is unphysical right from the start.

In Fig. 2(b) we consider the same dark cases as in Fig. 2(a) but for nearly vanishing base resistivity ($r_b = 0.01 \text{ m}\Omega\text{cm}^2$); this is motivated by the fact that in the present LR-$R_s$ theory only emitter and grid contribute to the series resistance (cf. [20, 21]). In this case, for not-too-high currents, there is a very good agreement between the LR-$R_s$ curve for $g = \frac{1}{2}$ (relevant for the rectangular grid geometry [20] as it is modeled here) and the Joule-$r_s$ curve. Thus, Fig. 2(b) shows that Eq. (1) is a very good approximation to describe the onset of the decrease of $R_s$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{(a) Lumped series resistance in dependence on the forward-bias diode current. Parameter: $r_e = 45 \text{ m}\Omega\text{cm}^2$, $r_{b,1} = 4.5 \text{ m}\Omega\text{cm}^2$, $r_{b,2} = 1.5 \text{ m}\Omega\text{cm}^2$, $r_{b,3} = 0.01 \text{ m}\Omega\text{cm}^2$, LR-$R_s,1$: $g = 3.8$; LR-$R_s,2$: $g = 1.8$, LR-$R_s,3$: $g = \frac{1}{2}$. (b) Systematic increase of $g$ for increasing base resistivity and constant $r_e = 45 \text{ m}\Omega\text{cm}^2$.}
\end{figure}

Besides the case of $r_b \approx 0$, Fig. 3(a) shows further Joule-$r_s$ curves, for the same $r_e$ as before, but with increased base resistivities of $r_{b,1} = 4.5 \text{ m}\Omega\text{cm}^2$ and $r_{b,2} = 1.5 \text{ m}\Omega\text{cm}^2$. For these, the corresponding $g$ values of the LR-$R_s$ curves that lead to a good match are 3.8 and 1.5, respectively. Together with the value used in Fig. 1(b), $r_b = 7.5 \text{ m}\Omega\text{cm}^2$, for which $g = 6$ was found, this gives a clear indication that $g$ increases for increasing $r_b$. Figure 3(b) collects data from calculations of some more cases, explicitly showing the systematic increase of $g$ with $r_b$. Thus, the contribution of the base to the total series resistance, which so far is not considered in the LR-$R_s$ concept, can explain the measured value of $g = 1.8$ reported in the Introduction.

4. Conclusions

The main conclusion is that the lumped series resistance of a solar cell can be fully understood as (i) being fully linear and (ii) its value being dependent only the dark current, with (iii) the parameter $g$ determining the latter dependency; neither from experiment nor from theory is there any need for a nonlinear or an asymmetric $R_s$. From this basic understanding it becomes immediately clear why in previous works, due to lack of knowledge about the general current dependence of $R_s$ (which is the one just on $l_0$), it was concluded that $R_s$ is asymmetric [8–11, 13]: In those measurements, in the dark case there was a large diode current, whereas in the illuminated case there was a small one. Obviously, this is sufficient for the series resistance to be different for these cases, explaining why in [8–11, 13] $R_{s,\text{dark}} < R_{s,\text{light}}$ was claimed. However,
by measuring \( R_{s,\text{light}} \) in the same high-slope region of the \( I-V \) curve where \( R_{s,\text{dark}} \) was determined, the same small \( R_s \) value will be obtained, and if it were possible to perform the dark measurement at rather low currents, a high \( R_s \) value would be obtained. Altogether, “\( R_{s,\text{light}} \)” and “\( R_{s,\text{dark}} \)” are just labels that were used due to the lack of knowledge about \( R_s(I_D) \), however there is no fundamental asymmetry for \( R_s \) w.r.t. the current flow direction [25].

That the dependence on \( I_D \) is indeed the only relevant current dependency can nicely be seen in the work of Turek [5]: In all the equations given there, \( I_{\text{ph}} \) and \( I_{\text{ext}} \) always enter in such a way that their combination can identically be replaced by \( I_D \). Thus, when looking into the existing literature, one needs to carefully check what is meant by a “current-dependent \( R_s \)” – since there are three different currents in a solar cell (photo-, dark-, and external current), this isn’t a unique notion. For example, it is not the case that (as claimed, e.g., in [26, 27]) due to the dependence of \( R_s \) on the photocurrent, the illumination intensity variation method (IIVM) to determine the series resistance (cf., e.g., [1, 4, 19]) would be invalid – on the contrary, since there is no sole dependence on the photocurrent but only the combined dependency of \( R_s \) on both \( I_{\text{ph}} \) and \( I_{\text{ext}} \) as discussed above, and since in the IIVM \( I_D \) is kept constant, this method is one of the best \( R_s \) determination methods available. Also, one needs to watch out how \( R_s \) is defined in the various literature works: A definition which is not in conformity with the general current dependence, \( R_s(I_D) \), may lead to unphysical artefacts (for example: \( R_s \) drops for small photocurrent in [17, Fig. 3] and at short circuit [17, Fig. 4]).

To come back to the starting point: For rightly declaring the lumped \( R_s \) of a solar cell to be a nonlinear quantity, one would need to prove this by rigorous measurements, avoiding all obvious nonlinearities of the solar cell, with the most obvious ones coming from the p–n junction and from the temperature. One has to remember that a solar cell is basically a network of different elements, so it is not straightforward to measure any of those elements alone; hence, to measure just \( R_s \), the p–n junction needs to be “passivated” – as it is the case, e.g., when the IIVM is performed at constant temperature. Only when operating conditions are chosen that are out of the bounds of standard operation of a solar cell, there are some hints that \( R_s \) isn’t symmetric [25], but this is an exception; under standard operating conditions, for constant \( I_D \) also \( R_s(I_D) \) is constant, which explicitly means that it is fully independent of \( I_{\text{ext}} \) and symmetric.

5. Summary and outlook

That \( R_s \) varies with the operation condition isn’t a higher-order effect but an intrinsic property of a distributed \( R_s \). All observed dependencies of the lumped \( R_s \) on the photocurrent or the external current can be traced back to the sole and general dependence on the dark current: The solar cell’s operating point is determined by \( I_{\text{ph}} \) and \( I_{\text{ext}} \), their combination [(cf. Eq (2))] determines \( I_D \), and from the latter, the value of \( R_s \) follows. This is fully in line with an older theoretical work stating the sole relevance of the forward bias for this effect [15]. This dependence on \( I_D \) (or, equivalently, on the forward bias) shows that \( R_s \) cannot be understood independently of the p–n junction’s switching state – which means that the standard equivalent circuit is insufficient because its basic idea is the full separation between diode and resistor. The LR-\( R_s \) concept leads to a slightly modified equivalent circuit explicitly containing the dark-current dependence of \( R_s \) [cf. Eq. (1)] that gives the onset of the decrease of \( R_s \) for not-too-large dark currents exactly; for larger currents, it is valid in very good approximation (cf. Fig. 2). This modified equivalent circuit contains the parameter \( g \) that determines at which \( I_D \) the decrease of \( R_s \) sets in; the value of \( g \) depends on the base resistivity. In the LR-\( R_s \) concept, the latter dependency wasn’t considered so far; the relevant theory will be presented elsewhere.

Data availability statement

All data shown in the Figures are obtained from numerical calculations in MATLAB. As such, the data can be reproduced by doing the numerical programming as outlined above. Please contact the main author (jwa@tf.uni-kiel.de) if you have difficulties reproducing them.
Author contributions
J.-M. Wagner: Conceptualization, Data Curation, Formal Analysis, Investigation, Methodology, Project Administration, Software, Visualization, Writing – Original Draft Preparation; J. Carstensen: Validation, Writing – Review & Editing; R. Adelung: Resources.

Competing interests
The authors declare no competing interests.

Acknowledgement
J.-M. Wagner acknowledges fruitful discussions with his former colleague A. Schütt (now with CELLOscan and AndyTRANS, Kiel, Germany, and also with Technische Hochschule Lübeck, Germany) and technical support from the Chair for Functional Nanomaterials (Prof. R. Adelung), Faculty of Engineering, Kiel University.

References


