



# Energy Utilization Factor of Solar Tower Systems

Indranil Paul<sup>1,\*</sup>  and Shireesh B. Kedare<sup>1</sup> 

<sup>1</sup>Department of Energy Science and Engineering, Indian Institute of Technology Bombay, India

\*Correspondence: Indranil Paul, paul.indranil@iitb.ac.in

**Abstract.** The solar tower system is one of the most promising concentrated solar power technologies that can address climate change issues by meeting energy demand sustainably while mitigating carbon emissions. The mirrors of the solar tower require a large share of the investment cost and occupy 4-8 times the ground compared to the mirror surface area. The share of the plant area cost may become significantly high for regions where the land is not cheap, especially in developing countries. In this study, a parameter named Energy Utilization Factor is introduced to quantify the utilization of the ground area in terms of available solar radiation energy. The variation of the Energy Utilization Factor is studied for a range of design variables without considering any specific dimensions for the heliostat and the receiver. The annual Energy Utilization Factor is seen to have a maximum value within 15 %, indicating the solar tower system is only capable of using one-seventh of the available solar resource on the ground. The maximum achievable Energy Utilization Factor is seen to increase with the tower height, but reduces with latitude. For a given heliostat field, the Energy Utilization Factor is seen to have a direct relationship with the amount of mirror area employed per unit ground area.

**Keywords:** Solar Tower, Energy Utilization Factor (EUF)

## 1. Introduction

In Solar Tower system the heliostat mirrors located on the ground reflect and concentrate the available solar radiation on a small absorber surface called the receiver. The receiver is located at an elevation from the ground called the tower optical height. The amount of energy concentrated on the receiver to the amount of solar energy available on the mirrors is termed as the optical efficiency. The overall performance and the economic feasibility of the solar tower system depend on the optical efficiency of the system.

The optical efficiency of a solar tower system depends upon various factors such as the receiver elevation, the arrangement of the heliostats in the ground, and the specifications of the heliostat and the receiver. The geographical location also affects the optical efficiency as the movement of the sun depends on the latitude. Several numerical investigations have been conducted for the maximization of the optical performance of the solar tower systems. Studies have been conducted to optimize the arrangement of the heliostats in the field for maximizing optical efficiency [1, 2, 3]. The arrangement of the heliostats generally follows a particular geometric pattern in these studies. Numerical investigation by Farges et al., 2018 revealed that, using a smaller sized heliostat while following a no-blocking heliostat arrangement criterion can improve the optical performance of the PS10 power plant [4]. Belaid et al., 2020 reported that, using smaller sized heliostats in the starting rows and utilizing gradually larger sized heliostats in the outer rows results in an improvement of the optical efficiency while reducing the land area requirement [5].

In these studies, one or more important design variables have been kept as constants. However, due to the inter-relationship between the design variables, the optical efficiency is not independent of those design variables that are kept constant. For example, the optimization of the size or shape of the heliostats should also take into account the dimensions of the receiver, as the intercept factor is dependent on both.

Carrizosa et al., 2015 conducted a numerical study to minimize the levelized cost of thermal energy of a solar tower system [6]. The heliostat field requires a large share of the overall investment cost, thus affecting the cost of energy. Along with that, the land area required for placing the heliostats also affects the cost of energy. The cost of land to the cost of mirror may vary from 1:20 to 1:100. For smaller values of the ratio, the share of the land cost in the overall investment cost remains very low. Therefore, the effect of the land cost on the cost of energy becomes negligible. But, for larger values of the ratio closer to 1:20, the land cost becomes a significant portion of the overall plant investment cost, thereby affecting the cost of energy produced. Therefore, optimal placement of the heliostats in the field to ensure maximum optical performance and proper utilization of the land area is important.

In this study, a new parameter named the Energy Utilization Factor (EUF) has been introduced which is defined as the ratio of energy reaching the receiver to the energy available on the ground area occupied by the heliostats. The distribution of the mirror area in the heliostat field is optimized to ensure maximum optical performance. The variation in the optimized Energy Utilization Factor has been studied for a number of design variables including the latitude and the tower height. Specific dimensions for the heliostat or the receiver aperture have not been considered and the heliostat mirror is represented with the help of mirror density.

## 2. System description

The receiver aperture has been considered to be the lateral surface of a cone and is represented by the tilt angle ( $\alpha$ ) from the vertical plane. The height of the aim point from the plane of the heliostat centers is called the Tower Optical Height ( $h$ ). The angle made by the reflected radiation from a heliostat with the receiver aperture normal is denoted by  $\phi$  in this work [7].

The field domain around the tower has been discretized into a grid of 100 m  $\times$  100 m elements. The amount of mirror area in each grid point is defined by the mirror density. The shading and the blocking efficiency are estimated using the mirror density following the work of Grigoriev et al. 2022 [8]. The shading (the blocking) efficiency depends on the amount of projected ground area towards the sun (the aim point) and the projected mirror area. The relation for the calculation of the shading and the blocking efficiency is given by Eq. (1-2).

$$\eta_{sh} = \sin \alpha_s / (\sigma \cdot \cos \theta) \quad (1)$$

$$\eta_{bl} = \sin \alpha_t / (\sigma \cdot \cos \theta) \quad (2)$$

where the angle of incident radiation with the mirror normal is denoted by  $\theta$ . The elevation angle of the sun and the elevation angle of the aim point from the heliostat are denoted by  $\alpha_s$  and  $\alpha_t$ , respectively. The mirror density ( $\sigma$ ) is defined as the amount of mirror area per unit ground area.

$$\sigma = \Delta A_m / \Delta A_g \quad (3)$$

### 2.1 Local optical efficiency

The instantaneous optical efficiency of an individual heliostat is the ratio of radiation focussed on the receiver aperture to the solar radiation intensity at that time instant. The instantaneous optical efficiency depends on the optical efficiency factors such as the reflectivity ( $\rho$ ), the cosine ( $\eta_{cos}$ ), the attenuation ( $\eta_{atn}$ ), the interception ( $\eta_{int}$ ), the shading ( $\eta_{sh}$ ), and the blocking ( $\eta_{bl}$ ).

$$\eta_{ins} = \rho \cdot \eta_{cos} \cdot \eta_{atn} \cdot \eta_{int} \cdot \eta_{sh} \cdot \eta_{bl} \quad (4)$$

In this work, an ideal situation is assumed with  $\rho = 1$ . The cosine efficiency is calculated from the dot product between the unit vector of the sun and the unit vector of the mirror normal. Using correlations given by Leary & Hankins, 1979 the attenuation loss can be predicted with sufficient accuracy based only on the slant range of the heliostat [9]. The interception efficiency is predicted with the help of the image spread in the vertical direction [7].

$$\eta_{int} = \cos \phi \quad (5)$$

where  $\phi$  is the angle made between the reflected radiation from a heliostat and the receiver aperture normal. The insolation un-weighted annual optical efficiency for a given field location is termed as the local optical efficiency in this work and is denoted by  $\eta_{loc}$ .

$$\eta_{loc} = \sum_1^{N_{ins}} \eta_{ins} / N_{ins} \quad (6)$$

## 2.2 Field optical efficiency

The annual energy concentrated on the receiver by a field location is the addition of the energy collected over all the time instants.

$$E_{ann,j} = (\eta_{loc,j} \cdot \sigma_j \cdot \Delta A_g \cdot I_b \cdot \Delta t) \cdot N_{ins} \quad (7)$$

where “j” denotes the index of the field location. The amount of ground area associated with the mirror density  $\sigma_j$  is denoted by  $\Delta A_g$ . The multiplication of  $\Delta A_g$  and  $\sigma_j$  indicates the amount of mirror area. The total annual energy concentrated on the receiver aperture by all the field locations is the addition of annual energy ( $E_{ann,j}$ ) of each location. The time interval between two instants of calculation is denoted by  $\Delta t$  and taken as 4 minutes in the study.

$$E_{ann,field} = \sum_{j=1}^{N_{loc}} E_{ann,j} \quad (8)$$

where  $N_{loc}$  is the number of field locations that constitute the heliostat field. The amount of solar radiation energy available on the mirrors over the year is given by Eq. (9).

$$E_{ann,avl} = \sum_{j=1}^{N_{loc}} (\sigma_j \cdot \Delta A_g \cdot I_b \cdot \Delta t \cdot N_{ins}) \quad (9)$$

The annual optical efficiency of the heliostat field is the ratio of the annual energy concentrated on the receiver over the year and the annual solar radiation energy available on the heliostat mirrors.

$$\eta_{field} = E_{ann,field} / E_{ann,avl} = \sum_{j=1}^{N_{loc}} (\sigma_j \cdot \eta_{loc,j}) / \sum_{j=1}^{N_{loc}} \sigma_j \quad (10)$$

where the field optical efficiency for the given heliostat field is denoted by  $\eta_{field}$ . For the calculation of insolation un-weighted field optical efficiency, the beam radiation intensity  $I_b$  is assumed to be constant for all time instants.

## 3. Methodology

The local optical efficiency of a field location depends on the mirror density, as the shading and the blocking efficiency are functions of it. To ensure the highest optical performance for a heliostat field, the optimal distribution of the mirror density is required. An optimization routine is developed to maximize the field optical efficiency for a given total mirror area ( $A_{mirror}$ ). For a given  $A_{mirror}$  the denominator of Eq. (10) is constant, resulting in the objective function:

$$OF: \max \sum_{j=1}^{N_{loc}} (\sigma_j \cdot \eta_{loc,j}) \quad (11)$$

### 3.1 Mirror density optimization

The input to the optimization routine is a matrix of local optical efficiency values, calculated for all the field locations in the domain at discrete intervals of mirror density. The optimization routine is divided into 2 segments – primary construction of the field, and iterative procedure of allocating mirror density. In the primary stage, the maximum local optical efficiency of each field location and the corresponding local mirror density is chosen. Following the ordering of the field locations with the maximum local optical efficiency, the number of field locations is determined to construct the required mirror area  $A_{mirror}$ .

In the iteration stage, the effect of the addition and the subtraction of the mirror density from the primarily selected field locations are evaluated. The iterations proceed in the following method:

1. Mirror density is added to field location which increases the numerator of Eq. (10) by the maximum extent
2. Mirror density is removed from the field location which reduces the numerator of Eq. (10) by the minimum extent

In each stage of the iteration, the mirror density is added to one field location and removed from another field location. In this way, the mirror area gets assigned to higher optical potential field locations from lower ones. After each iteration, the field optical efficiency is calculated for the new mirror density distribution.

The convergence of the optimization routine is dependent on two criteria. The primary criterion is the minimum improvement in the field optical efficiency, and the secondary is the minimum improvement in the average mirror density. The value of these parameters has been decided to be  $10^{-4}$  and  $10^{-2}$  respectively.

### 3.2 Energy Utilization Factor

The Energy Utilization Factor (EUF) can be defined as the amount of solar radiation energy reaching the receiver to the solar radiation energy available on a certain land area. In this work, EUF for an individual field location can be calculated from Eq. (12).

$$EUF_j = E_{ann,j} / (\Delta A_g \cdot I_b \cdot \Delta t \cdot N_{ins}) = \eta_{loc,j} \cdot \sigma_j \quad (12)$$

For the overall heliostat field associated with average mirror density ( $\sigma_{avg}$ ), the field average energy utilization factor ( $EUF_{field}$ ) can be determined using the following formulation.

$$EUF_{field} = E_{ann,field} / \sum_{j=1}^{N_{loc}} (\Delta A_g \cdot I_b \cdot \Delta t \cdot N_{ins}) = \eta_{field} \cdot \sigma_{avg} \quad (13)$$

## 4. Results and discussion

The local optical efficiency is a function of the latitude, the tower height, and the tilt angle of the aperture, along with the mirror density. Therefore, the optimized distribution of the mirror density and the corresponding EUF is dependent on the choice of these variables.

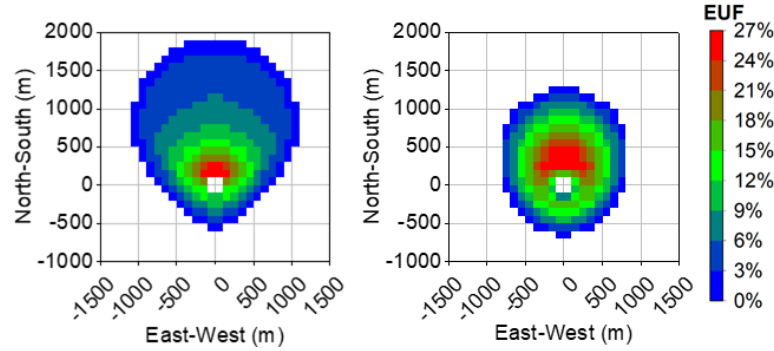
### 4.1 Spatial distribution of EUF

Parametric analysis and optimization are conducted at different tower heights, at latitude 20°N for  $A_{mirror} = 0.4 \text{ km}^2$ . The tilt angle of the aperture is varied from 0-90°. For a given tower height, the mirror density is optimized for each of the 91 tilt angle values. The optimum tilt angle for each tower height, the maximum field optical efficiency, the average mirror density of the optimized heliostat field, and the  $EUF_{field}$  are given in Table 1.

**Table 1.** Variation of field average  $EUF$  with the tower height for  $A_{mirror} = 0.4 \text{ km}^2$ , at latitude =  $20^\circ \text{N}$

Tower height (m)	$\alpha_{opt} (^\circ)$	$\eta_{field} (\%)$	$\sigma_{avg}$	$EUF_{field} (\%)$
50	6	63.25	0.0512	3.23
100	11	66.32	0.01	6.63
150	17	67.61	0.1324	8.95
200	22	68.44	0.1408	9.63
250	27	69.17	0.1492	10.32
300	32	69.82	0.1587	11.08

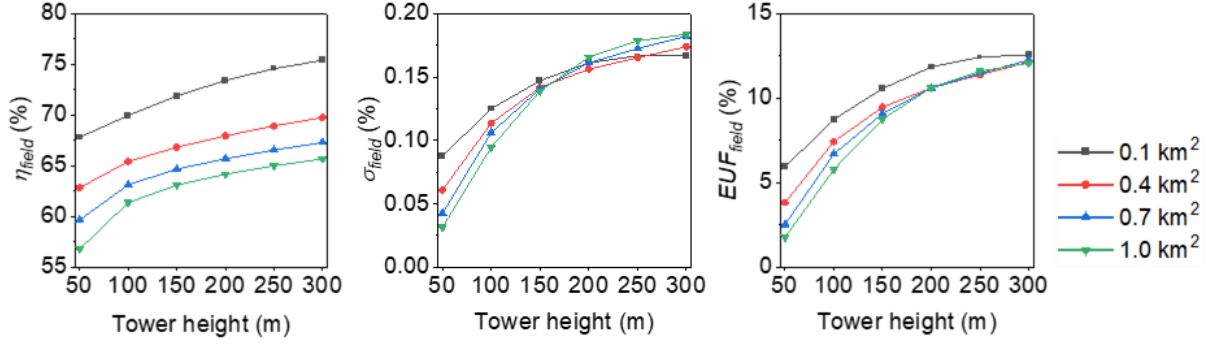
The spatial distribution of the  $EUF$  for individual field locations for tower heights 100 m and 300 m is shown in Fig. 1 for latitude  $20^\circ \text{N}$  and  $A_{mirror} = 0.4 \text{ km}^2$ . With increased tower height, the cosine efficiency improves, resulting in an overall increase in the local optical efficiency. With a taller tower, a greater amount of mirrors can be placed in field locations near the tower without hampering the blocking efficiency. Therefore, the heliostat mirror field becomes more compact with increased tower height as seen in Fig. 1. With increased average mirror density and an improved field optical efficiency, the field average energy utilization factor increases with the tower height.



**Figure 1.** Spatial distribution of  $EUF$  with tower height for  $A_{mirror} = 0.4 \text{ km}^2$ , at latitude =  $20^\circ \text{N}$ , for  $h = 100 \text{ m}$  and  $\alpha_{opt} = 11^\circ$  (left), and  $h = 300 \text{ m}$  and  $\alpha_{opt} = 32^\circ$  (right)

## 4.2 Parametric analysis of $EUF_{field}$

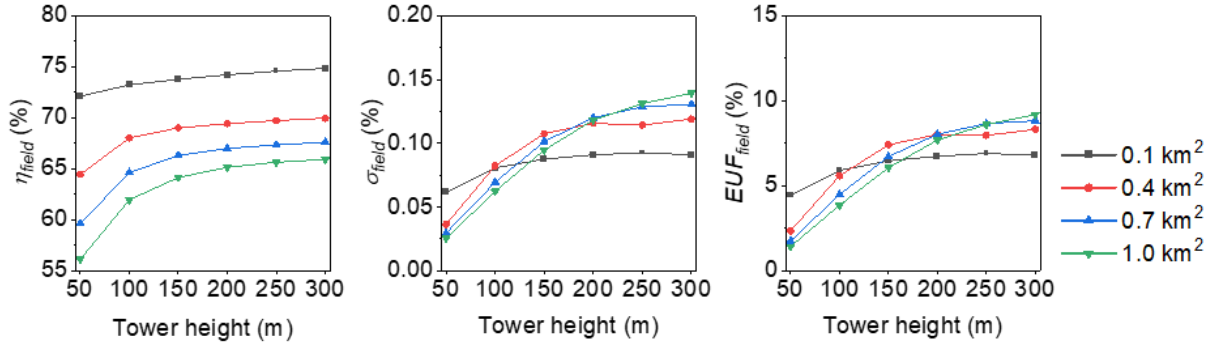
The field optical efficiency and the average mirror density are dependent on the optimum distribution of the mirror density, which in turn is affected by the latitude, the tower height, and the total mirror area. The variation of the  $EUF_{field}$  is studied for latitudes between  $0^\circ$  to  $50^\circ \text{N}$ , for  $A_{mirror} = 0.1 \text{ km}^2$  to  $A_{mirror} = 1 \text{ km}^2$ . Fig. 2 shows the variation of the field optical efficiency, the average mirror density, and  $EUF_{field}$  with the tower height at latitude  $10^\circ \text{N}$ . With increased tower height, the optical efficiency potential increases which permits the allocation of a greater amount of mirror area near the tower. As a result, the field optical efficiency and the average mirror density improve with the tower height. Consequently, the  $EUF_{field}$  is seen to increase with the tower height. For larger  $A_{mirror}$ , greater amount of mirror area is forced to occupy low optical efficiency field locations away from the tower, as a result, both the field optical efficiency and average mirror density are seen to decrease at a given tower height.



**Figure 2.** Variation on maximum field optical efficiency (left), average mirror density (middle) and field average EUF (right) with tower height for multiple  $A_{mirror}$ , at latitude  $10^\circ N$

Fig. 3 shows the variation of field optical efficiency, the average mirror density, and  $EUF_{field}$  with the tower height for different  $A_{mirror}$ , at latitude  $40^\circ N$ . With increasing latitude the cosine efficiency contours start to become biased towards the northern direction (south) for the locations in the northern hemisphere (southern hemisphere). At this comparatively high latitude, the high cosine efficiency field locations to the far north of the tower suffer from high shading and blocking loss, hindering the allocation of high mirror density. On the other hand, the optical efficiency potential combining cosine and blocking efficiency is much lower in the southern zone of the field, for  $40^\circ N$  compared to the same at latitude  $10^\circ N$ . The effect of low optical efficiency potential at higher local mirror density values results in lower average mirror density for the optimized configuration for higher latitude locations, as seen in Fig. 3.

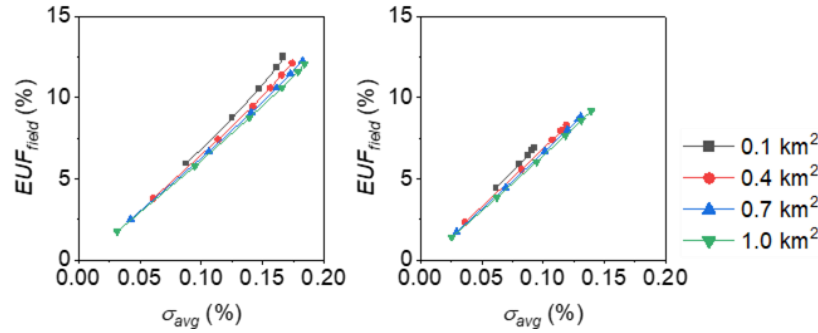
From Fig. 2 and Fig. 3, the difference in the field optical efficiency values is not significant between the two latitudes. However, a lower latitude location can achieve a high average mirror density for a given mirror area. Therefore, as depicted in Fig. 2 and Fig. 3 the average EUF for a given heliostat field area can be higher for a lower latitude location.



**Figure 3.** Variation on maximum field optical efficiency (left), average mirror density (middle) and field average EUF (right) with tower height for multiple  $A_{mirror}$ , at latitude  $40^\circ N$

#### 4.3 Marginal increase of $EUF_{field}$ vs $\sigma_{avg}$

Both the average mirror density and the energy utilization factor for an optimized heliostat field are seen to increase with the tower height. The optimum values of the average mirror density and the energy utilization factor for each tower height and field size combination are plotted in Fig. 4. Each curve in the graph consists of 6 scatter points corresponding to each tower height. The almost linear relationship between the energy utilization factor and the average mirror density is nearly independent of the latitude. Therefore, selecting the average mirror density for a heliostat field automatically determines the achievable energy utilization factor.



**Figure 4.** Marginal increase in the EUF with marginal increase of average mirror density at (a) latitude = 10 °N, and (b) latitude = 40 °N

## 5. Conclusion

The cost of the land area for a solar tower system may become significant in the overall investment cost based on plant location and land availability. Therefore, efficient utilization of the land area is of utmost importance. In this work, the mirror area distribution in the heliostat field is optimized to reach the maximum optical performance. A new performance parameter called Energy Utilization Factor (EUF) is introduced which indicates the portion of energy reaching the receiver from that is available on the ground occupied by the heliostat mirror field. Therefore, EUF indicates the proper utilization of the ground area in a solar tower system. The EUF of a field location is seen to have an inverse relationship with the distance from the tower in general, although it depends on the direction of the location with respect to the tower base. The field average value of EUF is seen to increase with a taller tower. The study is conducted for multiple latitudes by optimizing overall heliostat mirror areas between 0.1 km<sup>2</sup> to 1 km<sup>2</sup>. The EUF is seen to be higher for lower latitude locations than for higher latitudes, with the maximum value remaining within 15 %. As the outcome of the optimization, the average mirror density for a heliostat field is also determined. The EUF and the average mirror density are seen to be related almost linearly, irrespective of the latitude. Therefore, the proper utilization of the ground area is directly dependent on the average mirror density employed for the heliostat field of a solar tower system.

## Data availability statement

The data for this computational work can be requested via email to the corresponding author of this manuscript. Although it is the discretion and the unanimous decision of all the authors of this work to share the data of this work.

## Underlying or related material

Not applicable.

## Author contribution

Indranil Paul – Conceptualization, data curation, formal analysis, investigation, methodology, validation, visualization, writing – original draft

Shireesh B. Kedare – Methodology, project administration, resources, supervision, visualization, writing – review and editing

## Competing interests

The authors declare that they have no competing interests

## Funding

Not applicable

## References

- [1] F. J. Collado, and J. Guallar, A review of optimized design layouts for solar power tower plants with campo code. *Renewable and Sustainable Energy Reviews*, 20 (2013) 142-154. <https://doi.org/10.1016/j.rser.2012.11.076>
- [2] C. Noone, M. Torrihon, A. Mirsos, Heliostat field optimization: A new computationally efficient model and biomimetic layout, *Solar Energy*. 86(2) (2012) 792-803. <https://doi.org/10.1016/j.solener.2011.12.007>
- [3] S.M. Besarati, and D.Y. Goswami, A computationally efficient method for the design of the heliostat field for solar power tower plant, *Renewable Energy*. 69 (2014) 226-232. <https://doi.org/10.1016/j.renene.2014.03.043>
- [4] O. Farges, J.J. Bézian, and M. El Hafi, Global optimization of solar power tower systems using a Monte Carlo algorithm: Application to a redesign of the PS10 solar thermal power plant, *Renewable Energy*. 119 (2018) 345-353. <https://doi.org/10.1016/j.renene.2017.12.028>
- [5] A. Belaid, A. Filali, A. Gama, B. Bezza, T. Arrif, and M. Bouakba, Design optimization of a solar tower power plant heliostat field by considering different heliostat shapes. *International Journal of Energy Research*. 44(14) (2020) 11524-11541. <https://doi.org/10.1002/er.5772>
- [6] E. Carrizosa, C. Domínguez-Bravo, E. Fernández-Cara, and M. Quero, A heuristic method for simultaneous tower and pattern-free field optimization on solar power systems, *Computers & Operations Research*, 57 (2015) 109-122. <https://doi.org/10.1016/j.cor.2014.11.017>
- [7] I. Paul and S. B. Kedare, Determination of optically feasible heliostat field region for solar power tower system employing innovative receiver aperture, *Solar Energy*. 271 (2024) 112404. <https://doi.org/10.1016/j.solener.2024.112404>
- [8] V. Grigoriev, K. Milidonis, C. Corsi, and M. Blanco, Heliostat fields with a balanced mirror density, *Solar Energy*. 243 (2022) 336-347. <https://doi.org/10.1016/j.solener.2022.07.050>
- [9] P. L. Leary and J. D. Hankins, User's guide for MIRVAL: a computer code for comparing designs of heliostat-receiver optics for central receiver solar power plants, (1979) (No. SAND-77-8280). Sandia National Lab.(SNL-CA), Livermore, CA (United States).